LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2024**

18UMT3MC02 – VECTOR ANALYSIS AND ORDINARY DIFF. EQUATIONS

Date: 08-11-2024 Dept. No. Max. : 100 Marks

Time: 09:00 am-12:00 pm

SECTION A

Answer ANY FOUR of the following **(4 x 10 = 40)**

1. Prove that for any vector $\vec{A}$, $\nabla\times(\nabla\times\vec{A})=\nabla(\nabla.\vec{A})-\nabla^2\vec{A}$.
2. Find the directional derivative of $xy+yz+zx$ at the point $(1,1,3)$ in the direction of the vector $\vec{i}+2\vec{j}+2\vec{k}$.
3. Verify Stokes' theorem for $\vec{A}=(2x-y)\vec{i}-yz^2\vec{j}-y^2z\vec{k}$ taken over the upper half of the surface of the sphere $x^2+y^2+z^2=1, z\geq 0$ and the boundary curve C, the circle $x^2+y^2=1, z=0$.
4. Find the value of the integral $\int_C \vec{A}.\vec{dr}$, where $\vec{A}=yz\vec{i}+zx\vec{j}-xy\vec{k}$ if C is the curve whose parametric equations are $x=t, y=t^2, z=t^3$ drawn from $O(0,0,0)$ to $Q(2,4,8)$.
5. Solve the homogenous differential equation: $x\,dy-y\,dx=\sqrt{x^2+y^2}\,dx$.
6. Solve $\frac{dy}{dx}=\frac{x+2y-3}{2x+y-3}$.
7. Find the general solution of the differential equation $(D^2+2D+5)y=xe^x$.
8. Select suitable method and solve the differential equation $3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x$.

SECTION B

Answer ANY THREE of the following **(3 x 20 = 60)**

9. (a) If $\nabla\varphi=(y+y^2+z^2)\vec{i}+(x+z+2xy)\vec{j}+(y+2zx)\vec{k}$ and if $\varphi(1,1,1)=3$, find φ.

   (b) Evaluate $\iiint_V \nabla.\vec{F}\,dV$ if $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ and if V is the volume of the region enclosed by the cube $0\leq x,y,z\leq 1$.

10. Verify the divergence theorem for $\vec{A}=(x+y)\vec{i}+x\vec{j}+z\vec{k}$ taken over the region V of the cube bounded by the planes $x=0, x=1, y=0, y=1, z=0, z=1$.

11. Using Green's theorem, show that $\int_C (3x^2-8¿y^2)dx+(4y-6xy)dy=20¿$, where C is the boundary of the rectangular area enclosed by the lines $x=0, x=1, y=0, y=2$ in the xoy plane.

12. (a) Solve: $(1-x^2)\frac{dy}{dx}+2xy=x\sqrt{1-x^2}$, given that $y=0$ when $x=0$.

    (b) Find the solution of $\frac{dy}{dx}-y\tan x=\frac{\sin x\cos^2 x}{y^2}$.

13. Solve: $\left(D^2+16\right) y = 2e^{-3x} + \cos 4x$.

14. Using variation of parameters, solve $\frac{d^2 y}{dx^2} + y = sec\, x$.

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